10.12 Flat Slabs

Beam and slab construction has the advantage of providing intermediate supports to the slabs, thus reducing the effective span of the slabs. But the beams require larger depths thus leading to more heights of the buildings. In some situations, specially in warehouses, it is desirable to have larger clear ceiling heights. Flat slabs are the slabs which rest directly on the columns without beams and thereby provided a larger clear ceiling height for the same given total height of a building. In addition, the form-work requirement is also reduced when compared with the beam and slab construction. Flat slabs are invariably two-way slabs and rest on several columns. Sometimes the top of the columns are widened so as to provide wider base to support the slab. Such widened portions are called *column heads*. There is a limit to which one can treat the widened portion as a part of the column. The width must be limited to the portion within 90° of the segment as shown in Figs. 10.14b and c. Any projection beyond the column head should really be treated as a part of the slab rather than that of the columns.

In other words, it should be treated as thickening of the slab at the column head. Such thickened portions of the slab are called *drops*. The drops are sometimes known as capital of the columns. The drop when provided should be rectangular or circular and need to be about one-third of the panel length.

In each panel, the slab is divided into column and middle strips in each direction. The width of the *column strip* is equal to half of the column spacing and it is placed half on either side of the column line. In case of unequal spans, it can be taken equal to half of the average. In addition, it should also be restricted to 0.5 times the column spacing in any direction. *The middle* strip is the one bounded by the column strips and its width is equal to the spacing of the columns minus the width of the column strip. The width is usually equal to or greater than half of the spacing of the columns.

Let b_{ci} = width of column strip in the ith column row

b_{mi} = width of the middle strip

L_{xi} = spacing of the columns in the x-direction in the ith panel

L_{yi} = spacing of the columns in the y-direction in the ith panel

Then width of the column and middle strips spacing in the x-direction is given by

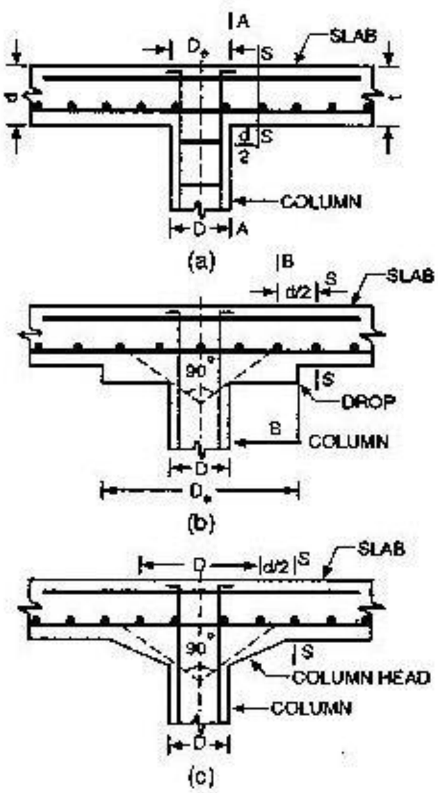


Fig. 10.14 Typical Flat Slabs.

Table 10.4 Distribution of BM Across the Panel

	Strip and its boundary conditions	Per cent of the total BM
1.	*Column strip*	
	a) Negative BM at exterior support	100
	b) Negative BM at the interior support	75
	c) Positive BM	60
2.	*Middle strip*	
	The difference between the panel moment and the column strip moment.	

$$b_{ci} = 0.25\ (L_{yi-1} + L_{yi}) < 0.25\ (L_{xi} + L_{xi+1})$$

$$b_{mi} = L_{yi} - 0.25\ (L_{yi} + L_{yi} + 1)$$

Similarly, for the widths of the strips in the y-direction. In most situations, the spacing of the columns in one direction is same in all the panels. Therefore, the calculation of the widths of the strips is normally a trivial exercise.

The slabs can be analysed as equivalent frames having idealized continuous wall supports along the transverse column lines. The stiffness of the column is divided by the panel width, and is considered as the stiffness of the vertical element per unit width of the frame. The analysis has to be done in both the directions independently as two sets of independent frames. Such an idealization introduces undue bending moments into the middle strip; therefore, the moments and shear forces computed by this method must be proportioned with higher weightage to the column strip when compared with that of the middle strip. The analysis is to be carried by loading only three-fourths of the total live load in each panel and full dead load. However, in case of mat foundation slabs, full load coming from the column should be taken as the load, and also no reduction should be given to liquid loads. The frames should be analysed for two load conditions, namely all panels loaded and alternative panels loaded. The critical section for design of moment is the section at the face of the column or at the face of drop. The critical section for shear force design is at peripheral line around the column at a distance 0.5d from the face of the column or the face of the drop. Usually the thickness of the drop is taken large enough to eliminate the failure of the section around the periphery of the column. The section and the reinforcement must be designed to withstand the weighted proportioned moment as given in Table 10.14.

10.13 Direct Method of Limit Analysis of Flat Slabs

The collapse mode in flat slab is due to negative yield lines along the column line and positive yield lines along the mid-span line. The total slab can be treated as resting as a continuous support in each direction with width equal to the width of the slab. The three critical sections in the outer span are shown in Fig. 10.14 as *AA*, *BB* and *CC*. The resisting moment capacity at section *AA* across the column heads is more than that at *BB*. The failure mechanism of the drop *D*, such that the yield line at *BB* is avoided.

Let M_{rp} = total positive moment capacity at the middle line

$M_{r\,nc}$ = total negative moment capacity at the face of the column or drop

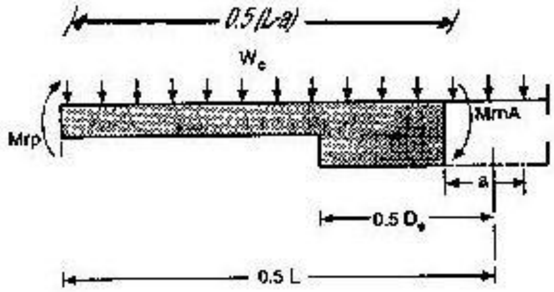


Fig. 10.15 Critical Moments in Segment of a Flat Slab.

B = width of the panel
L = span of the panel
a = column size
D_c = drop size

Figure 10.15 illustrates the average resisting moment capacities along with the other dimensions of a segment between the yield lines. The equilibrium of forces about the face of the column line gives

$$(M_{rp} + M_{r\,nA}) = 0.5\,(L - a)\,B\,(0.5)\,\frac{(L-a)}{2}\,w_c$$

or
$$M_{rp} + M_{r\,nA} = \frac{B(L-a)^2}{8}\,w_c \tag{10.74}$$

The above equation gives the sum of the magnitudes of the positive and negative bending moment capacities which is approximately equal to the maximum BM on a simply supported beam of span of (L—a). One can select a relation between the positive and the negative moment capacities and then establish the desired sections. Let

$$M_{r\,nA} = c_A M_{rp} = c_A M_r \tag{10.75}$$

then Eq. (10.74) gives

$$M_r = \frac{B(L-a)^2\,w_c}{(1 + c_A)} \tag{10.76}$$

Consider another possible case of yield line generating at section BB instead of at AA. Let

$M_{r\,nB}$ = moment capacity at BB

The equilibrium equation of the segment CB gives

$$M_{rp} + M_{r\,nB} = \frac{B(L - D_c)^2}{8}\,w_{c2} \tag{10.77}$$

The collapse load based on the first and second mechanisms can be expressed as

$$w_{c1} = \frac{B(M_{rp} + M_{r\,nA})}{B(L-a)^2} \tag{10.78}$$

$$w_{c2} = \frac{8(M_{rp} + M_{mB})}{B(L - D_c)^2} \quad (10.79)$$

The condition for the first mode of failure is to occur,

$$w_{c1} < w_{c2}, \text{ that is}$$

$$(M_{rp} + M_{mA})\,(L - D_c)^2 < (M_{rp} + M_{mB})\,(L - a)^2 \quad (10.80)$$

$$\text{or} \quad L - D_c < (L - a)\sqrt{\frac{M_{rp} + M_{mB}}{M_{rp} + M_{mA}}}$$

$$\text{or} \quad D_c > L - (L - a)\sqrt{\frac{M_{rp} + M_{mB}}{M_{rp} + M_{mA}}} \quad (10.81)$$

The fraction under the square root will be less than one and it will be in the range of 0.7. The first mode of failure is likely to occur if,

$$D_c \geq (0.16L + 0.85a) \quad (10.82)$$

Assuming a is in the range of $L/15$, the width of the drop to be maintained for the first mode of failure is sure to occur if,

$$D_c \geq 0.25L \quad (10.83)$$

There are situations when the first mode of failure is likely to occur even if D_c is in the range of $0.20L$. The design of the flat slab is best illustrated through examples.

Interior span: In the interior spans, three yield lines will be formed; one at each face of the column lines and the other one at the mid-span. The corresponding collapse load relation can be obtained as

$$M_{rl} = \frac{B(L - a)^2\, w_c}{8(1 + c_B)} \quad (10.84)$$

where $M_{mB} = c_B M_r$

The design procedure consists of computing the average moment capacity required from the collapse load Eq. (10.76). Then the BMs on the column and middle strips can be obtained by assigning appropriate weightages as given in Table 10.4. The overall depth of the slab is controlled by the bending moment in the exterior span of the column strip. The depth of the slab is first designed and then the reinforcements required at various places are computed.

Some of the design considerations are:

1. The end span should not be larger than the interior spans.
2. The ratio of the successive span lengths should be within 0.75 to 1.33.
3. A cantilever projection of about one-third of the exterior span can be permitted with appropriate modification in the bending moments.
4. The design live load should not be more than three times the dead load. This is a major constraint, and when ignored it may effect the magnitude of the positive bending moment.

The sum of the magnitudes of the positive and average of the negative bending moments is equal to

$$M_0 = \frac{WL_0}{8} \quad (10.85)$$

where M_0 = sum of the magnitudes of the positive and average of the negative bending moments, and it is $M_0 = M_3 + 0.5\,(M_1 + M_2)$ (10.86)

M_3 = positive bending moment

M_1 and M_2 = magnitudes of negative bending moments at the face of the two columns on either side of the span (vide Fig. 10.16)

L_0 = clear span extending from face to face of the columns or capitals. It should be equal to larger of the two unequal adjacent spans and > 0.65 spacing of the columns.

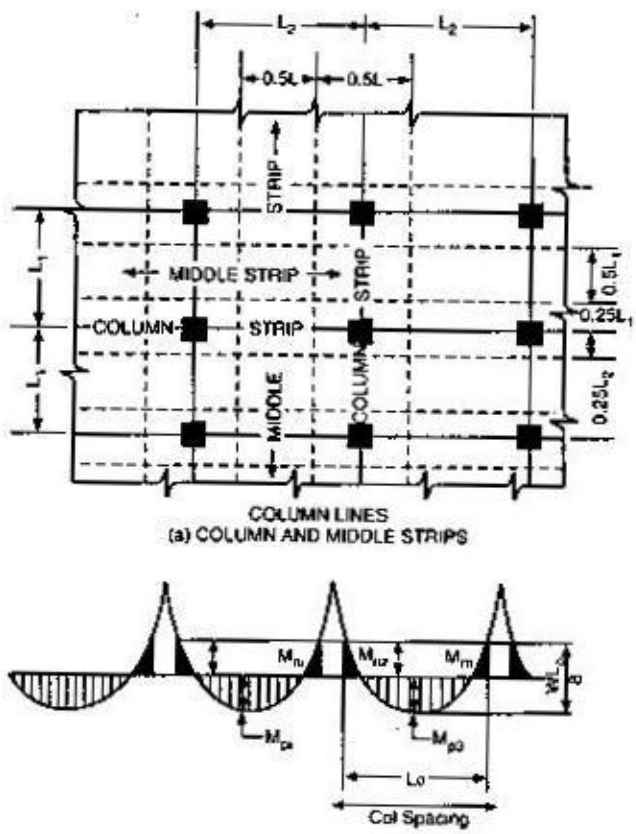
W = design load on the clear span = $w_c B\,(L - a)$

The relative magnitudes of the negative and positive bending moments can be calculated using the ratios given in Table 10.5

The magnitude of the bending moment can be obtained as

$$M_t = c_i M_0 \qquad (10.87)$$

where the moment coefficients, c_i's are listed in Table 10.5.



(a) COLUMN AND MIDDLE STRIPS

(b) TYPICAL BENDING MOMENT

Fig. 10.16 Notation in Flat Slab.

Table 10.5 Relative Values of Positive and Negative Bending Moments BM

	Bending moment and location	c_i
1.	*Interior spans*	
	a) Negative BM = M_{ni}	0.65 M_0
	b) Positive BM = M_{pi}	0.35 M_0
2.	*End span*	
	a) Exterior negative BM = M_{n1}	0 to 0.1 M_0
	b) Positive BM = M_{p3}	0.60 M_0
	c) Interior negative BM = M_{n2}	0.70 M_0

The distribution of the moments at a section between the column and middle strips should be as per Table 10.4. The value of c_B to be used in the equation is 0.65/0.35 = 1.85.

For convenience let $c_B = c_0$

or $c_A = c_0$ depending the failure mode.

Design for shear: The slabs are likely to fail by diagonal tension around the column faces rather than along a section parallel to the column line. The critical section for shear is the peripheral plane which is at a distance 0.5d from the face of the column or the column head or drop. In case there is an opening near this zone, appropriate deduction in the peripheral length should be made proportional to the distance of the critical section to that of the opening from the centre of the column.

Let B_0 = size of the opening

d_1 = distance of the opening from centre of the column

$0.5(a + d)$ = distance of the critical section from the centre of the column

Then the ineffective width to be deducted from the length of the critical section is given by

$$b_d = \frac{B_0(0.5)(a + d)}{d_1} \tag{10.88}$$

the length of the critical section for shear is

$$b_0 = 4(a + d) - b_d \tag{10.89}$$

Typical notation for BM, etc. The subscripts refer to:

c = column strip
m = middle stirp
n = negative BM
p = positive BM
1 = exterior critical section usually at the face of the exterior columns or drops
2 = interior column face of the exterior panel
3 = middle point in the exterior panel
i = interior panels

Figure 10.16 illustrates the notations.

M_{n1} = negative BM at section 1 on the panel
M_{nc1} = negative BM at section 1 on the column strip

M_{pi} = positive BM at the interior panel

M_{pci} = positive BM at the interior panel on the column strip

The other notations are defined similarly.

10.14 Design Examples of Flat Slabs

EXAMPLE 10.6 *Flat roof slab without column heads*: A roof slab is supported on columns spaced at 5 m apart in two perpendicular directions. The size of the square column is 440 mm and the live load on the roof is 1500 N/m^2. The load of the waterproof treatment course on the slab is 2000 N/m^2. Design a flat slab without drops or column heads. Height of the column above the mat foundation is 6 m.

Design data

Spacing of the columns $L_1 = L_2 = L = B = 5$ m

Size of the column $a = 0.44$ m

Live load $w_l = 1.5$ kN/m^2

Superimposed load $w_s = 2.0$ kN/m^2

$f_{ck} = 20$ MPa and $f_y = 415$ MPa

The limit state strength coefficients are $K = 0.138$, $j = 0.80$.

Design of the section for moment: The design is done by using direct design method. For the purpose of estimating the self-weight of the slab, let thickness of the slab be assumed in the range of $L/20$ for the sales without column heads.

Let the thickness of the slab $t = 0.24$ m

Self-weight = 0.24(25) $= w_g = 6.0$ kN/m^2

Total dead load $w_d = w_g + w_s = 8.0$ kN/m^2

Clear spacing between the columns is

$$L_0 = L - a = 5 - 0.44 = 4.56 \text{ m}$$

The total design load in a panel is

$$W_c = w_c L L_0 = \gamma_f (w_d + w_l)\, L L_0 = (1.5)9.5(5)(4.56) = 324.9 \text{ kN}$$

Sum of the magnitudes of the bending moments in the panel between the faces of the columns

$$M_0 = \frac{w_c L_0}{8} = \frac{324.9(4.56)}{8} = 185.193 \text{ kNm} = 0.185193 \text{ MNm}$$

Magnitude of the negative BM at the face of the columns in the interior panels is

$$M_{ni} = 0.65\, M_0 = 0.12038 \text{ MNm}$$

The column is resting on a mat foundation; therefore, the base of it can be treated as fixed. The end walls of the hall restrain the free lateral movement of the roof slab; therefore, the top of the column can be assumed as fixed in position and rotation. Hence the effective height of the column is

$$L_c = 0.65\, H = 0.65(6) = 3.9 \text{ m}$$

The exterior negative and positive bending moment coefficients and the interior negative bending moment coefficients are taken from Table 10.5 and are:

$$c_1 = 0.1,\ c_3 = 0.60,\ c_2 = 0.70$$

The corresponding bending moments are:

$$M_{n1} = c_1 M_0 = 0.01852 \text{ MNm}$$

$$M_{p3} = c_3 M_0 = 0.11112 \text{ MNm}$$

$$M_{n2} = c_2 M_0 = 0.12964 \text{ MNm}$$

The bending moments on the column strip are computed from the total moment after applying the distribution factors given in Table 10.4. They are:

$$M_{nc1} = M_{n1} = 0.01852 \text{ MNm}$$

$$M_{pc3} = 0.6\ M_{p3} = 0.06667 \text{ MNm}$$

$$M_{nc2} = 0.75\ M_{n2} = 0.09723 \text{ MNm}$$

$$M_{ncl} = 0.75\ M_{ni} = 0.090285 \text{ MNm}$$

The thickness of the slab is constant and without any column heads or drops. The maximum moment occurs at the interior column face of the exterior panel. The effective depth of the slab is given by

$$d = \sqrt{\frac{M_{nc2}}{bKf_{ck}}} = \sqrt{\frac{0.09723}{2.5\,(0.138)\,(20)}} = 0.12 \text{ m}$$

Use $d = 0.15$ m and $t = 0.18$ m.

The slab was assumed to be 0.24 m for the purpose of computing self-weight. Since the thickness actually provided is 0.18 m, the moments are revised.

$$w_g = 0.18\,(25) = 4.5 \text{ kN/m}^2$$

$$w_t = 4.5 + 2 + 1.5 = 8.0 \text{ kN/m}^2$$

The actual value of w_t is $(8/9.5) = 0.842$ times the load that was assumed. Therefore, the final moments are computed by multiplying the previous moments by 0.842. The design moments are (all in MNm):

$$M_0 = 0.842\,(0.185193) = 0.15593$$

$$M_{n1} = 0.1\ M_0 = 0.015593;\ M_{p3} = 0.09356$$

$$M_{n2} = 0.10916,\ M_{nc1} = M_{n1} = 0.015593$$

$$M_{pc_3} = 0.05614,\ M_{nc2} = 0.08187$$

$$M_{nci} = 0.07602,\ M_{pci} = (0.35)(0.75)(M_0) = 0.04093$$

Normally no shear reinforcement is provided in the slabs. It is, therefore, desirable that the adequacy of the depth of the section against shear should be checked at the earliest.

The shear strength for diagonal tension failure is

$$\tau_c = 0.25\sqrt{f_{ck}} = 1.12 \text{ MPa}$$

The critical shear plane is the peripheral plane which is at a distance $0.5d$ from the face of the column. The length of the critical section is

$$b_0 = 4\,(a + d) = 4\,(0.44 + 0.15) = 2.36 \text{ m}$$

The shear force on the plane is

$$V_c = w_t\,(L_1\,L_2 - (a+d)^2)\,\gamma_f$$
$$= 8.0(25 - 0.64^2)\,(1.5) = 295 \text{ kN}.$$

The nominal shear stress is

$$\tau_v = \frac{V_c}{b_0 d} = \frac{0.295}{(2.36)(0.15)} = 0.83 \text{ MPa}$$

The nominal shear stress is less than the shear capacity of the concrete; therefore, there is no need for transverse reinforcement or thickening of the slab.

Design of reinforcement

Column strip: The positive BM at mid-span of exterior panel is

$$M = M_{pc3}$$

The area of the tension steel at the bottom at the mid-span is

$$A_{st3} = \frac{1.15 M_{pc3}}{jdf_y} = \frac{(1.15)\,56140}{(0.80)(0.15)(415)} = 1296 \text{ mm}^2$$

Provide 12 numbers of ϕ12 bars in the column strip at the mid-span.

$$A_{st} \text{ (provied)} = 1356 \text{ mm}^2$$

Half of the bars are to be cranked.

The negative BM at exterior support $M = M_{nc1}$

The area of the reinforcement at top near the column line is

$$A_{st1} = \frac{1.15 M_{nc1}}{jdf_y} = \frac{(1.15)\,15593}{(0.80)(0.15)(415)} = 360 \text{ mm}^2$$

The minimum area of the reinforcement required is

$$A_{stm} = \frac{0.12\,bt}{100} = 540 \text{ mm}$$

Needed: 5 numbers of 12 mm bars in the 2.5 m width of the column strip at top near the column line (reinforcement as required). The cranked bars from positive reinforcement are adequate. The positive BM in the column strip of the *interior panel* is

$$M_{pci} = (0.35)\,(0.75)\,M_0 = 0.04093 \text{ MNm}$$

The area of the tension reinforcement required is

$$A_{sti} = \frac{(1.15)40930}{(0.80)(0.15)(415)} = 945 \text{ mm}^2$$

Needed: 9 numbers of ϕ12 bars at the bottom ($A_{st} = 1017 \text{ mm}^2$) of which four bars are cranked. The negative bending moment at the interior column of the end panel is

$$M_{nc2} = 0.08187 \text{ MNm}$$

The area of the tension reinforcement required is

$$A_{st_2} = \frac{(1.15)\,81870}{(0.80)\,(0.15)\,(415)} = 1891 \text{ mm}^2$$

Extra: 6 numbers 12 mm bars at top over the column line ($A_{st} = 1921 \text{ mm}^2$).

Design of middle strip: The bending moment in the middle strip are obtained by subtracting the BM on the column strip from the total BM on the panel. One can calculate the number of bars required by simple proportion of the bending moments subjected to the minimum reinforcement.

1. The negative BM at the exterior support is zero; however, a nominal reinforcement is provided at top.

Needed: 5 *numbers of* ϕ12 bars at top in the middle strip at the edge.

2. The positive BM in the exterior panel

$$(0.35)\,(0.25)\,(M_o) = 0.01364 \text{ MNm}$$

The area of the reinforcement needed for this moment is 314 mm^2, and it is less than the minimum required. So *provide* 5 *numbers of* ϕ12 at the bottom.

It can be seen that the bending moments in the middle strip are only nominal and the minimum tension reinforcement governs the design. This minimum reinforcement is 5 nos. of 12 mm bars in 2.5 m width. The reinforcement details are given in Fig. 10.17. The curtailment and cranking of the bars is recommended as per the normal practice.

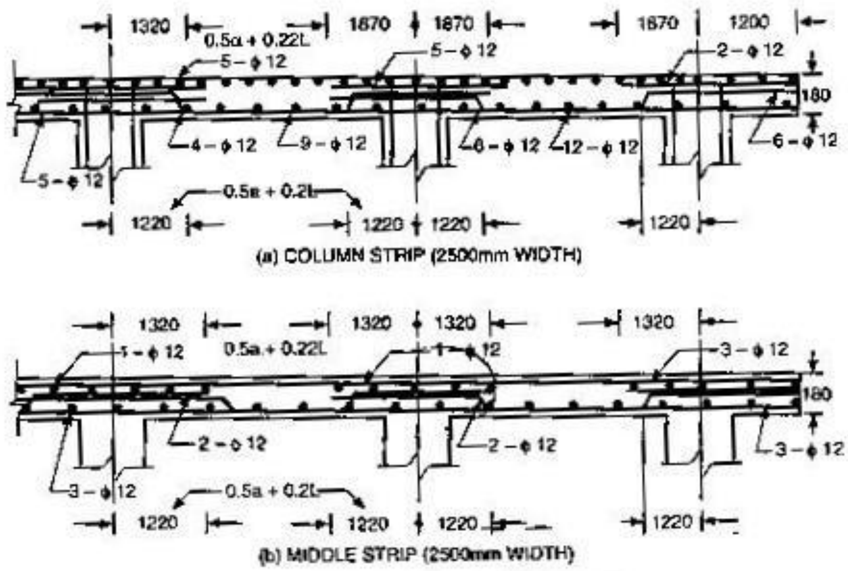


Fig. 10.17 Reinforcement Details in Flat Slabs of Example 10.6.

Development length is given by

$$L_d = \frac{\phi f_y}{4\,\tau_{bd}} = \frac{12\,(415)}{4\,(1.2)\,(1.6)} = 648 \text{ mm}$$

EXAMPLE 10.7. *Flat slab with column heads.* A slab is supported on columns spaced at 5 m apart in both directions. The sizes of the column and column head are 440 by 440 mm and 760 by 760 mm respectively. The superimposed live and dead loads are 1.5 and 2 kN/m^2 respectively. The height of the column above the mat foundation is 6 m, including 160 mm high column head.

Table 10.6 Reinforcement Details (all bars of 12 mm dia) (Example 10.6)

	No. of bars and length		Location	Strip
1.	2,3740	extra	Over the exterior column line and at top face	Column
2.	5,3740	extra	Over the interior columns line of the end panel and at top face	"
3.	6,5000 6,7070	straight cranked	At the mid-span at bottom face in the the end way	"
4.	5,5000 4,7070	straight cranked	At the mid-span at bottom face in the interior bay	"
5.	3,2640	extra	Over the exterior column line and at top face	Middle
6.	1,2640	extra	Over the interior column line and at top face	"
7	3,5000 2,6520	straight cranked	At the mid-span and at bottom face	"

Design data

Spacing of the columns = $L_1 = L_2 = L = B = 5$ m

Size of the column = $a = 440$ mm

Size of the column head = $a_1 = 760$ mm

$w_s + w_l = 3500$ N/m^2

$f_{ck} = 20$ MPa, $f_y = 415$ MPa

The design coefficients are: $K = 0.138$, $j = 0.80$.

Design of the section. For the purpose of estimating the self-weight of the slab, let the overall thickness of the slab be assumed in the range of $L/20$.

Net thickness of the slab = $t = 0.24$ m

Self-weight = $w_g = 0.24(25) = 6$ kN/m^2

The total dead load = $w_d = 6 + 2 = 8$ kN/m^2

The total design load = $w_t = 8 + 1.5 = 9.5$ kN/m^2

Clear spacing between the column heads is

$$L_0 = L - a_1 = 5 - 0.76 = 4.24\ m \nless 0.65L$$

The total design load in a panel is

$$W = \gamma_f w_t L L_0 = (1.5)(9.5)(5)(4.24) = 302.1 \text{ kN}$$

The sum of the magnitudes of the positive and negative bending moments in a panel is

$$M_0 = \frac{WL_0}{8} = \frac{302.1\,(4.24)}{8} = 160.113 \text{ kNm}$$

The magnitude of the negative BM at the face of the column head in the interior panel is

$$M_{ni} = 0.65\,M_0 = 0.65\,(160.113) = 10.4.07 \text{ kNm}$$

The coefficients of exterior negative and positive BMs, and the interior bending moment are taken from Table 10.5; they are: $c_1 = 0.10$, $c_3 = 0.60$, $c_2 = 0.70$

The corresponding bending moments are:

$$M_{n1} = c_1 M_0 = 0.016 \text{ MNm}$$

$$M_{p3} = c_3 M_0 = 0.09606 \text{ MNm}$$

$$M_{n2} = c_2 M_0 = 0.1121 \text{ MNm}$$

The bending moments on the *column strip* of 2.5 m width at different locations are computed using Table 10.4. These moments are:
Negative BM at the exterior face of the exterior panel:

$$M_{nc1} = M_{n1} = 0.016 \text{ MNm}$$

$$M_{pc3} = (0.6) M_{p3} = 0.05763 \text{ MNm}$$

The negative BM in the interior column face of the exterior panel is :

$$M_{nc2} = 0.75\ M_{n2} = 0.08408 \text{ MNm}$$

The negative BM in the interior panels is

$$M_{nci} = 0.75\ M_{ni} = 0.07865 \text{ MNm}$$

The values listed are only the magnitudes of the moments and the thickness of the slab is governed by the largest magnitude of the BMs. The effective depth of the slab needed is given by

$$d = \sqrt{\frac{M}{Kbf_{ck}}}\ \sqrt{\frac{M_{nc2}}{Kbf_{ck}}}\ \sqrt{\frac{0.08408}{0.138(2.5)(20)}} = 0.120 \text{ m}$$

Use $d = 0.14$ m, then the overall thickness of the slab is

$$t = 0.14 + 0.03 = 0.17 \text{ m}$$

The thickness of the slab was assumed as 0.24 m for the purpose of self-weight; therefore, the earlier moment computed is on the safer side.

Self-weight $w_g = 0.17\ (25) = 4.5 \text{ kN/m}^2$

$$w_t = 4.5 + 2.0 + 1.5 = 8.0 \text{ kN/m}^2$$

Design for shear. The shear strength for diagonal tension is

$$\tau_c = 0.25\sqrt{f_{ck}} = 1.12 \text{ MPa}$$

The critical shear plane is the peripheral plane which is at a distance $0.5d$ from the face of the column head. The total length of the critical shear plane section is

$$b_0 = 4\ (a_1 + d) = 4\ (0.76 + 0.14) = 3.6 \text{ m}$$

The total shear force on this plane is

$$V_c = \gamma_f\, w_t\, (L_1 L_2 - (a_1 + d)^2)$$

$$= (1.5)(9)(25 - 0.90^2) = 326.56 \text{ kN}$$

The nominal shear stress is

$$\tau_v = \frac{V_c}{b_0 d} = \frac{326560}{2600\ (140)} = 0.65 \text{ MPa}$$

The nominal shear stress is within the allowable value.

Design of reinforcement

Column strip: The positive BM at mid-span = M_{pc3} = 0.05763 MNm and the area of the reinforcement needed is

$$A_{st3} = \frac{1.15 M_{pc3}}{jdf_y} = \frac{(1.15)57630}{(0.80)(0.14)(415)} = 1426 \text{ mm}^2$$

Provides 13 numbers of 12 mm bars and crank 6 bars from each side. Then the actual steel provided is A_{st} = 1469 mm^2. The negative BM at the exterior support is M_{nc1} and the area of the reinforcement which is to be laid at the top near the column is

$$A_{st1} = \frac{1.15\, M_{nc1}}{jdf_y} = \frac{(1.15)\,16000}{(0.80)(0.14)(415)} = 396 \text{ mm}^2$$

6 bars are already cranked, so add two extra bars.

The negative bending moment at the interior of the end panel is M_{nc2} = 0.08408 MNm, and the area of the reinforcement is

$$A_{st2} = \frac{(1.15)84080}{0.80(0.14)(415)} = 2080 \text{ mm}^2$$

Provide 13 *numbers of* 12 mm *bars at top* (extra).

A_{st} provided is 2147 mm^2 (inclusive of 6 cranked bars from each direction)

Design of middle strip. The bending moments in the middle strip are obtained by subtracting the bending moments of the column strip from the total bending moments in the panel at the corresponding section. The minimum reinforcement at any section can be taken as 0.12% of the area of the concrete, and is:

$$A_{smin} = \frac{0.12\, bt}{100} = \frac{0.12(2500)(170)}{100} = 510 \text{ mm}^2$$

Therefore, the minimum number of 12 mm bars at any given section is 5. The bending moment on the middle strip is two-thirds of that of the column strip in the positive BM zone and it is one-third of the negative BM zone. It can be observed that the amount of the reinforcement needed in the middle strip is governed by the minimum requirement rather than by the bending moments. Figure 10.18 illustrates the reinforcement details. The development length of the bars is

$$L_d = \frac{\phi f_y}{4\,\tau_{bd}} = \frac{12(415)}{4(1.2)(1.6)} = 691 \text{ mm}$$

Comment about the desirability of column heads or drops: The provision of column heads or drops decrease the effective span and consequently the bending moment. Any decrease in the concrete due to smaller thickness of the slab is usually compensated by the addition of the concrete at the column heads and extra formwork. The length of the cranked bars or top bars is increased in the case of slabs with column heads, so the gain in the decrease of the area of the reinforcement is compensated. The drops or column heads become almost unavoidable if the nominal shear stress exceeds the allowable value. In general, such a situation arises when (i) the spacing of the columns is large, say, more than 6 m, (ii) the size of the column is very small compared with the panel size, say, the size of the column is less

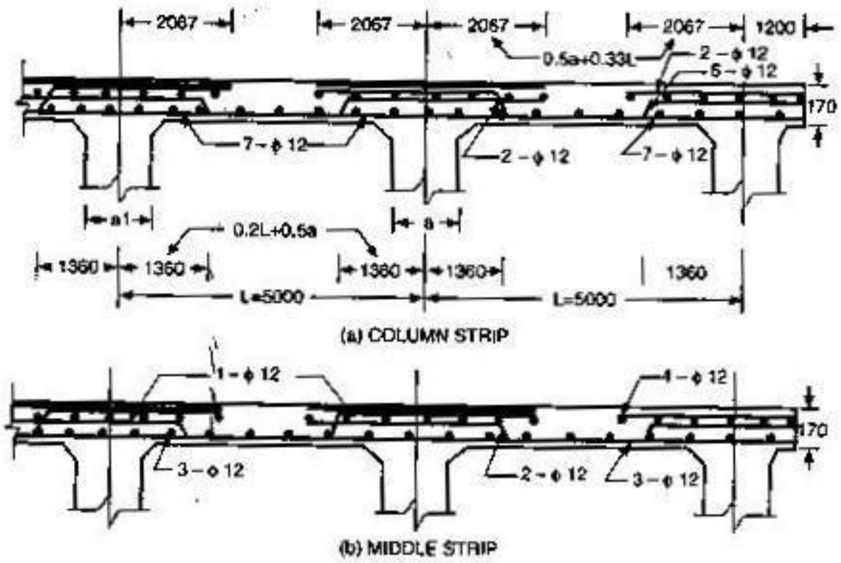


Fig. 10.18. Reinforcement Details of Flat Slab of Example 10.7.

than $L/15$; and (iii) the total and intensity on the slab is high, say, it is more than 30 kN/m^2. Otherwise, it is desirable to design the flat slab without column heads which is not only economical but helps in aesthetic and functional aspects also.